turbulent boundary layers; δ^{**}, δ_t^{**}, magnitude of the momentum and energy losses; ρ_0, w_0, T_0, density, velocity, and temperature at the outer boundary-layer limit; ρ_W, T_W, density and temperature on the permeable wall; w_W, velocity of injected gas delivery; $\bar{j} = \rho_W w_W / \rho_0 w_0$, blowing intensity; $\omega = w/w_0$; $\vartheta = (T_W - T)/(T_W - T_0)$, dimensionless velocity and temperature; w', T', velocity and temperature pulsations (fluctuations); Ψ_{SX}, relative heat-exchange coefficient; b_{tx}, permeability parameter; b_{cr}, critical permeability parameter; St_{0x}, Stanton number under standard conditions; ΔP, static pressure drop on the permeable wall and in the free stream; y, transverse coordinate; Re_x, Reynolds criterion; c_f, friction coefficient.

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STATIONARY TWO-DIMENSIONAL ROLLING WAVES ON A VERTICAL FILM OF FLUID

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The wave characteristics of two-dimensional stationary rolling waves on a vertical film of fluid are investigated experimentally using the electrodiffusion and shadow methods.

In order to calculate with sufficient accuracy the processes of heat and mass exchange during the drainage of thin films of fluid, it is important to be able to take into account the influence of wave formation, especially in the case in which the process rate is determined by mass exchange with respect to the gas [1].

In spite of the large number of theoretical papers written on this area of study [2-5], there is as yet no certainty that the theoreticians' predictions about the physics of wave formation on the surface of thin vertical layers of fluid are correct.

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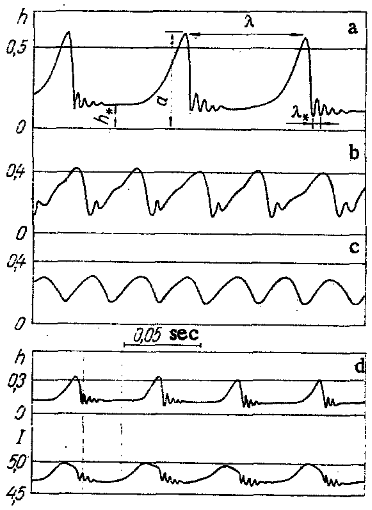

Fig. 1. Oscillograms of the profiles of excited waves (a, b, and c) and the simultaneous recording of friction and thickness at a single point (d); a) c = 0.5 m / sec; b) 0.38; c) 0.3 m / sec; h, mm; I, μA.

The overwhelming majority of the theoretical models are based on an assumption about the purely capillary nature of wave formation, the two-dimensional structure of waves, and the smallness of the wave amplitude compared with the mean film thickness.

The only experimental investigation in which the difficulty of organizing two-dimensional waves is pointed out and specially organized two-dimensional waves are actually observed was carried out by Kapitsa and Kapitsa [6]. In other experimental papers such as [7-9], as a rule, no differentiation is made between two-dimensional and three-dimensional waves, and the analysis and comparison with theoretical results of these data are, consequently, made much more difficult.

It is also a well-known fact that waves on the surface of vertical films of fluid grow rapidly and at a very short distance their amplitude can exceed the mean film thickness, at which point so-called rolling wave conditions are realized. The rolling waves have not been studied in previously published papers and the results of the present authors' experiments indicate that the characteristics of large-amplitude waves on a thin vertical film are heavily dependent on the viscosity of the fluid and may be noncapillary in nature. This possibility has been demonstrated theoretically for slightly nonlinear waves in [4].

The aim of this paper is to establish the fundamental laws governing the behavior of stationary two-dimensional waves of random amplitude on a vertically draining thin film of fluid.

The working section of the experimental apparatus is a transparent plastic tube 1 m long and 60 mm in diameter with a film of fluid draining off along its external surface. The thickness of the film is measured by the shadow method and the friction at the wall, by the electrodiffusion method. A diagram of the apparatus and the procedure for measuring the friction and thickness are described in detail in [10] and [11]. The amplitudes, wavelengths, and mean film thickness are measured from the oscillograms of the instantaneous film thickness. The wave velocity is determined from the phase shift between two simultaneously recorded fluctuations in the thickness which correspond to two different points along the tube. The film-thickness measurement error does not exceed 7%, the wavelength measurement error does not exceed 9%, and the phase-velocity measurement error does not exceed 8%.

When a fluid film drains off along a vertical wall, waves are generated at a certain distance from the fluid outflow point, which increase rapidly in amplitude and reach a stationary mode in which the phase velocity of wave propagation, the wavelength, and the amplitude do not vary or vary slightly along the tube. In all the modes investigated by the present authors the stationary wave forms correspond to Fig. 1. The length of the two-dimensional wave zone is, unfortunately, short and does not exceed 150 mm. Thereafter the waves decay and become three-dimensional. Kapitsa and Kapitsa [6] have excited waves artificially by fluctuations in the fluid flow rate in order to regulate the two-dimensional waves and increase the region over which they exist.

In these experiments the flow rate is fluctuated by oscillating a membrane in a chamber connected by rubber tubing to the input device of the working section. The whole system is filled with fluid. The membrane is activated by an electric motor with a number of revolutions which can be regulated by a crank gear. The membrane oscillation frequency varies from a few hertz to a few dozen hertz. The frequency of motor revolution is measured directly in the course of the experiment.

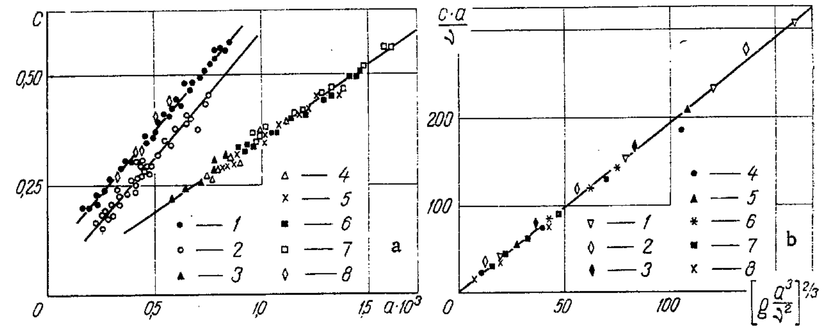

Fig. 2. Dependence of phase velocity of waves on their amplitude: a – excited waves: 1) glycerine solution $\nu = 2.16 \cdot 10^{-6}$ m^2/sec, $\sigma/\rho = 65.2 \cdot 10^{-6}$ m^3/sec^2; 2) alcohol solution $\nu = 2.12$, $\sigma/\rho = 28.5$; 3) glycerine solution $\nu = 11.2$, $\sigma/\rho = 55.9$, Re = 4.0; 4) Re = 5.9; 5) 7.9; 6) 9.8; 7) 12.45; 8) natural waves: glycerine solution $\nu = 2.16$, $\sigma/\rho = 65.2$; c, m/sec; a, m; b – excited waves: 1) water $\nu = 0.9 \cdot 10^{-6}$ m^2/sec, $\sigma/\rho = 72 \cdot 10^{-6}$ m^3/sec^2; 2) water $\nu = 1.03$, $\sigma/\rho = 72.9$; 3) aqueous solution of glycerine and alcohol $\nu = 1.65$, $\sigma/\rho = 46.8$; 4) alcohol solution $\nu = 2.12$, $\sigma/\rho = 28.5$; 5) glycerine solution $\nu = 2.16$, $\sigma/\rho = 65.2$; 6) $\nu = 3.9$, $\sigma/\rho = 60.7$; 7) $\nu = 7.2$, $\sigma/\rho = 57.6$; 8) $\nu = 11.2$, $\sigma/\rho = 55.9$.

When waves are excited on the film surface by fluctuations in the fluid flow rate, the wave pattern takes the same form as in the case of natural drainage. The waves are, however, generated directly at the input, and the region of two-dimensional well-formed waves is significantly larger than in the case of natural drainage.

Excited regular stationary waves are generated in a certain frequency range dependent on the physical properties and flow rate of the fluid. For example, for an aqueous solution of glycerine with a viscosity $\nu = 3.9 \cdot 10^{-6}$ m^2/sec and a surface tension of $\sigma/\rho = 60.7 \cdot 10^{-6}$ m^3/sec^2 when Re = 20 the excited waves exist in 7-32-Hz range of frequencies. Beyond the limits of this range there is no regular wave pattern.

In the case of the maximum possible excitation frequency, low-amplitude waves are generated with a form close to the sinusoidal (Fig. 1c). As the frequency is reduced the waves take on the characteristic appearance of the rolling wave (Fig. 1a, b, and d) with high-frequency oscillations preceding a steep front.

The fact that with a given fluid flow rate the wave characteristics of the stationary wave process are wholly determined by the frequency of the oscillations superimposed on it and are independent of the amplitude of fluctuations in the flow rate is interesting and extremely important from the point of view of the methodology of the experiment. This circumstance has been demonstrated by special experiments to measure the amplitude of fluctuations in the friction at the wall in the annular distributing slit from which the film flows out and at a certain distance from the rim of the slit in the region of which the stationary-wave mode exists; the film thickness is also measured here.

It should be noted that the wave characteristics of natural waves for a given fluid are dependent only on the flow rate, and when the frequency of the superimposed oscillations coincides with the frequency of the natural waves the wave patterns are identical, which has been observed in specially run experiments.

Water and aqueous solutions of ethyl alcohol and glycerine are used as the working fluids with the viscosity varying from $0.9 \cdot 10^{-6}$ to $11.2 \cdot 10^{-6}$ m^2/sec and the surface tension varying from $28.5 \cdot 10^{-6}$ to $72.9 \cdot 10^{-6}$ m^3/sec^2. The range of variations in the Reynolds number Re = Q/ν, where Q is the fluid flow rate relative to the pipe perimeter, is 4-40.

Figure 2a shows the dependence of the phase velocity of rolling waves on the amplitude a for several fluids with different Re numbers. The amplitude a is defined as the maximum film thickness. A linear relationship is clearly observed in all the experiments, with both natural and excited waves, between the phase velocity of the rolling waves and their amplitude. The phase velocity is not dependent on the mean fluid flow rate and is slightly dependent on surface tension so that the results can be correlated in the form of the relation (Fig. 2b)

$$\frac{ca}{\nu} = 1.98 \left(g \frac{a^3}{\nu^2}\right)^{2/3}, \tag{1}$$

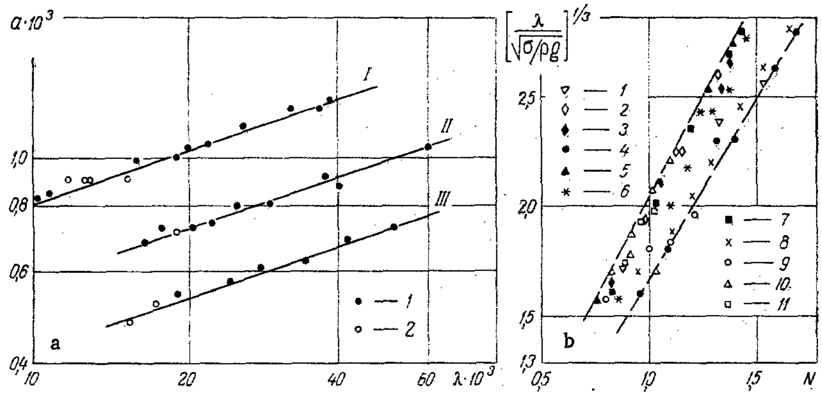

Fig. 3. Connection between wave amplitude and wavelength: a – glycerine solution, $\nu = 7.2 \cdot 10^{-6}$ m²/sec, $\sigma/\rho = 57.6 \cdot 10^{-6}$ m³/sec²; I) Re = 20; II) 10.1; III) 6.1 [1) excited waves; 2) natural]; a, m; λ, m; b – for notation 1–8 see Fig. 2b; 9, 10, 11 denote natural waves for fluids 4, 5, and 7, respectively (see Fig. 2b). $N = 1/(\mathrm{Re}^{0.46}\mathrm{Fi}^{0.02})\sqrt[3]{ga^3/\nu^2}$.

where the ca/ν complex is the Reynolds number plotted in terms of phase velocity and wave amplitude and $g(a^3/\nu^2)$ is the Galileo number. The relationship (1) is best represented for practical use in the form

$$c = 1.98\left(\frac{g^2}{\nu}\right)^{1/3} a. \tag{2}$$

All the data on stationary excited waves are contained in this relationship.

The wave amplitudes in these experiments are great and the ratio of maximum film thickness a to mean thickness varies within limits of 1.1 and 3.0.

Figure 3a depicts data for a water–glycerine solution in terms of the dependence of the amplitude of the rolling waves on the wavelength for different Re numbers. The dependences for all other fluids take an analogous form. By using the experimental relationshp $a \sim \lambda^{1/3}\mathrm{Re}^{0.46}$, obtained from these graphs, it is possible to represent all the data on wavelength for both natural and excited waves (with a maximum scatter of 20%) in the form of Fig. 3b and to describe them approximately by the following relation:

$$\left[\frac{\lambda}{\sqrt{\sigma/\rho g}}\right]^{1/3} \sim \left(g\frac{a^3}{\nu^2}\right)^{1/3} \frac{1}{\mathrm{Re}^{0.46}\,\mathrm{Fi}^{0.02}}, \tag{3}$$

where $\mathrm{Fi} = (\sigma/\rho)^3/g\nu^4$ is the film number. The range of variations in the film number Fi for the fluids used in the experiments is $1.18 \cdot 10^6$–$5.82 \cdot 10^{10}$.

The rolling waves shown on the oscillograms in Fig. 1a, b, and d can be subdivided into a large principal wave and a forerunner in the form of a damped harmonic wave moving in front of it with the same velocity c. The wavelength λ_* is then greater than the local mean thickness of the fluid layer. The dependence of the rate of propagation of the forerunner on the wavelength λ_* is used to prove its capillary nature.

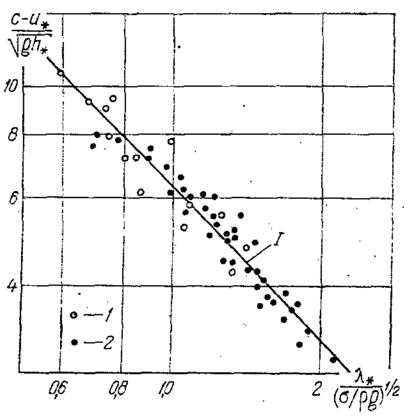

Fig. 4. Rate of propagation of capillary waves: 1) alcohol solution, $\nu = 2.12 \cdot 10^{-6}$ m²/sec, $\sigma/\rho = 28.5 \cdot 10^{-6}$ m³/sec²; 2) glycerine solution, $\nu = 7.2$, $\sigma/\rho = 57.6$; I) formula (4).

It is known that the rate of propagation of capillary long waves in shallow water is defined as

$$c - u_* = \frac{2\pi}{\lambda_*} \sqrt{\frac{\sigma}{\rho} h_*}, \quad (4)$$

where u_* is the fluid layer velocity, λ_* is the wavelength, and h_* is the fluid layer thickness. Since a shallow ripple is propagated through the residual layer of thickness h_* (Fig. 1a and d), the velocity of the surface h_* can be determined from Nusselt's formula for a smooth laminar film:

$$u_* = \frac{3}{2} \cdot \frac{g h_*^2}{3\nu}. \quad (5)$$

In fact, as can be seen from the simultaneous recordings of the instantaneous thickness of the film and the limiting diffusive current I, related to the magnitude of the friction τ at the wall by the formula $\tau = AI^3$ [10, 11], where A is the calibration coefficient, the flow of the residual layer (marked off by the dashed lines in Fig. 1d) is purely laminar.

Experimental data for glycerine and ethyl alcohol solutions are compared in Fig. 4 with the theoretical relationship (4). A good agreement between experiment and theoretical calculation is observed for all the fluids used in the experiments, which proves that the structure of the shallow ripple in front of the wave is capillary.

NOTATION

h, film thickness, m; a, wave amplitude, m; c, phase velocity of waves, m/sec; λ, wavelength, m; I, limiting diffusive current, μA; u, fluid velocity, m/sec; g, acceleration of gravity, m/sec^2; ν, coefficient of kinematic viscosity, m^2/sec; σ, coefficient of surface tension, kg/sec^2; ρ, fluid density, kg/m^3; Q, specific flow rate of fluid, m^2/sec; τ, friction at wall, N/m^2; $Re = Q/\nu$, Reynolds number; $Fi = (\sigma/\rho)^3/g\nu^4$, film number; *, value relative to residual layer.

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